Ways of Comprehending

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This paper is the Prologue of the forthcoming book *Ways of Comprehending,* by A.S. Fokas (World Scientific, 2022): This book is dedicated to my three children and to all young people in the hope that it will offer them the happiness and personal fulfilment that follows from acquiring an understanding of the origin of their thoughts, feelings, and actions.

The search for understanding gives rise to deep admiration for the immense wisdom and beauty of Nature, and in particular for its greatest achievement: the human brain. Writing this book, I felt deep sense of gratitude for the privilege of being able to enjoy a plethora of complex and multifaceted creations of Nature and humanity. I hope, and expect, that those who read this volume will experience similar feelings.

'Unification' and 'analogical thinking' are central themes of this book. In this connection, it is worth noting that the completion of the formalism that unifies the four fundamental forces of nature, the gravitational, electromagnetic, weak, and strong interactions, still stands as the holy grail of physics. By analogy, it is natural to attempt to integrate the biological and cultural 'forces' dictating life. In this volume, an effort is made to explore this unification.

By analysing fundamental neuronal mechanisms, it will become clear that the human brain is predisposed to seek *knowledge* and *beauty*, without the artificial distinction between *sciences* and *humanities*. It is argued that such a grand quest requires an interdisciplinary, integrative approach. Furthermore, it is suggested that this search is facilitated by employing the notions of *cognition*, *computability*, *creativity*, and *culture*. The necessity for such a unified approach follows from the insight that *everything is related to everything else*. Perhaps no one expressed this fact better than Leonardo da Vinci, the embodiment of interdisciplinarity:

> 'Study the science of art. Study the art of science. Develop your senses—especially learn how to see. Realize that everything connects with everything else'.

A crucial part of an interdisciplinary approach to knowledge and culture is the appreciation that life generously provides many sources of pleasure and satisfaction, beyond the utilization, efficiency, power, and beauty of technological creations. Indeed, in life there also exists that which, according to Ludwig Wittgenstein, 'cannot

be said'. It is the part which is not only non-verbal, but more generally non-algorithmic. It is *spiritual*, where spirituality refers to accessibility of emotions and other creations of the unconscious. It is the *transcendental* part that lives in the exceedingly rich and mysterious world of the unconscious. I believe that life is incomplete without this part of insight, beauty and potential eudemonia.

A Paradox and its Resolution

Our times are characterized by an apparent paradox: on the one hand, there exists a vast amount of information and resources, and therefore the potential for broad knowledge, understanding, and personal fulfilment. On the other hand, there is a strong tendency for specialization and limited appreciation of the enormously rich scientific and cultural achievements of humanity. Moreover, the continuous bombardment of the brain with discrete, disconnected pieces of information brings confusion and a sense of alienation. The feeling of living in a world that becomes more and more difficult to comprehend. If we do not understand the world we live in, how can we define the meaning and the value of our actions?

The above paradox brings to mind Erwin Schrödinger's passionate appeal to fulfil the human 'longing for unified, all-embracing knowledge'. This statement, made in his highly influential 1944 book *What is Life*,[1] was followed by the Nobel Laureate's expression of regret that '[…] it has become next to impossible for a single mind fully to command more than a small specialized portion' of the existing vast amount of knowledge. According to Schrödinger, the only way out of the above formidable difficulty is 'that some of us should venture to embark on a synthesis of facts and theories, albeit with second-hand and incomplete knowledge of some of them'.

I fully embrace the significance of the above appeal. Recognizing that this problem is indeed unsurpassable, especially since general knowledge has grown immensely in the last fifty years, I will attempt a less ambitious project. My aim is to present a proper *framework capable of approaching knowledge and understanding in the unified, all-embracing manner* envisioned by Schrödinger. This approach has become possible as a result of the remarkable progress achieved since the turn of the millennium regarding the *functioning of the brain*. For example, it will be shown that there now exist appropriate tools for elucidating the mechanisms responsible for the human 'quest for unification,' noted by Schrödinger. My attempt to delineate the above approach has been motivated by the privilege of a broad education in engineering, mathematics, physics, and medicine, as well as my exposure to a wide range of areas through research activity and published work.[2]

[1] E. Schrödinger, *What Is Life? The Physical Aspect of the Living Cell* (Cambridge University Press, 1944).

[2] From differential equations and the Riemann zeta function, to symmetries and geometry; from the general theory of relativity, to particle physics; from protein folding and chronic myelogenous leukaemia,

The grand illusion

Answering the question 'what is consciousness?' is considered one of the most important open problems in the history of sciences. Many deep scholars have written extensively on the subject, approaching it from a variety of angles, including neuroscience, artificial intelligence, philosophy, mathematics, and physics. It has even motivated works of literature.[3] As discussed in the last chapter of this volume, I believe that this problem *can* be solved, provided that it is correctly defined and is placed in the proper perspective. In my opinion, this requires the acceptance of the fact that *unconscious and conscious processes form a continuum*, along with the *primacy of the unconscious*. In this connection, motivated by the analysis of a plethora of neurophysiological studies, including the detailed scrutiny of visual perception presented in chapter 4, I postulate that

every conscious experience is preceded by an unconscious phase.

Let me clarify the critical importance of unconscious processes by using a simple example. My wife, Regina, enters my study in our house in Cambridge. As a result of intricate processes that are *entirely* unconscious, *my brain* perceives Regina. I will refer to the *unconscious construction* of this percept as the *mental representation* of Regina. About a third of second later, *my brain informs me of what it already knows*. Namely, my unconscious informs my consciousness of the presence of my wife. At this moment, the miracle of awareness takes place: I perceive Regina. I will refer to the *conscious construction* of this percept the *mental image* of Regina. As soon as unconscious processes begin to construct the mental representation of my wife, these processes simultaneously form infinitely many *associations* related to her. Consequently, my unconscious decides that I should greet Regina with the statement 'Good morning, my love'. About a third of a second later, it instructs my consciousness to implement this decision. My wife's unconscious perceives my greeting, and then, it informs her consciousness; etc.

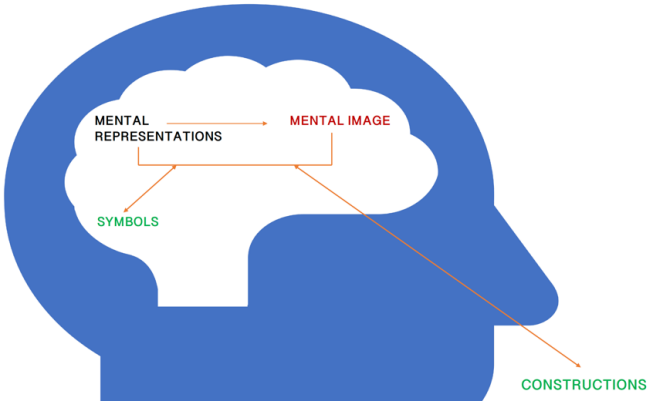
Mental images (3D holograms) are continuously updated, but they are always 0.3–0.5 of a second *behind* reality. Consequently, the interaction with the external environment in 'real time' is, paradoxically, the task of the unconscious and *not* of consciousness. Indeed, it is intricate *unconscious* processes that allow us to walk, to drive without crashing, to hit or catch a ball, to see glimpses of colors, etc. We erroneously believe that our *conscious self* is in charge of our communications, actions, feelings, etc. In reality, our consciousness is nothing but the 'faithful assistant' of the unconscious. I refer to this remarkable, but largely unappreciated fact, as the 'grand illusion'. Following the groundbreaking work of the neurophysiologist Benjamin

to mathematical models for *C. elegans* and for aspects of Covid-19; from medical imaging and 'deep learning', to philosophy and the quantification of fractality in paintings of Piet Mondrian.

[3] T. Stoppard, *The Hard Problem: A Play* (Grove Press, 2015).

Libet (which will be discussed in a future volume), this illusion has been discussed with regards to 'decision making' (free will).[4] However, as evident from the above example, the fact that awareness *always* lags a considerable amount of time behind unconscious processes, underlines *every* mental function of *any* organism possessing consciousness. In this sense, the label 'grand illusion' is, hopefully, well justified.

In contrast to our evolutionary predecessors, we have the privilege of possessing language. Undoubtedly, this enormously enriches our capacity to communicate. For example, instead of the statement 'Good morning, my love', animals must use a variety of indirect ways to communicate their emotions. As a result of the transformative impact of language, many scholars have highlighted this great gift as the *key* difference between us and other creatures possessing consciousness. In my opinion, this is *not* entirely correct. It is argued in this volume that our qualitative advantage in comparison to other animals is our predisposition to *construct real versions* of our mental images and our mental representations, *or to assign to them specific symbols.* I will label the emerging constructions or symbols *re-representations.* In addition to language, the re-representations of mathematics, computers, technology, and arts, are of crucial importance for the development of our culture. Regarding the arts, as noted in chapter 9, many great artists have explicitly stated that their creations often begin in their unconscious. This is consistent with my assertion that the origin of re-representations is not only mental images, but also mental representations. The above relationships between mental representations, mental images, and re-representations are indicated in the diagram below.



As a mathematician I have greatly benefited from this unique capacity of the human brain. Indeed, the thought process of a mathematician often compels them to *write* specific mathematical formulas. As soon as a formula is written, a dynamic interaction begins between this specific re-representation and related unconscious processes. This hugely generative process finally expresses itself with another

[4] D.M. Wegner, *The Illusion of Conscious Will*. Foreword by D. Gilbert. Introduction by T. Wheatly (The MIT Press, New Edition, 2017).

formula, etc. Generalizing this particular case, it becomes evident that the continuous and multilevel interaction between a plethora of re-representations and unconscious processes is the essence of the astounding human creativity.

The first 'big bang' in biological evolution was the emergence of consciousness, or more precisely, of the defining property of consciousness, which (as Libet pointed out) is *awareness*. In my opinion, the second 'big bang' was the emergence of re-representations. This is unique to humans; it constitutes, in fact, the birth of humanity. As a result of the unlimited richness of re-representations, the interaction of humans with their environment became far more complex than that of their evolutionary predecessors. To emphasise the highly dynamic and everchanging nature of this interaction, I will use the term *heterodynamics*. This also conveys the qualitative difference between this situation and the quasi-static state that characterises the internal environment of living organisms, called *homeostasis*. Heterodynamics, which is decisively affected by re-representations, dictates cultural evolution, and for humanity, this is far more important that biological evolution.

How do we comprehend?

The earlier postulate that unconscious processes are a necessary condition for the creation of a conscious experience implies that *anything* we do, think, and feel, is crucially affected by the 'echo' of processes of which we are completely unaware. In addition to the implications of this postulate for 'ways of comprehending', my further suggestions in this direction are based on a second postulate:

> *gaining deep insight necessitates the need to decipher fundamental biological, and especially neuronal, mechanisms*.

Such an analysis implies that the following notions are *indispensable tools for the comprehension of our thoughts, feeling, and actions, and more generally for the search for a hidden reality*:

> 'continuity', 'associations', 'interconnectedness', 'analogical thinking', 'continuity', 'abstraction', 'generalization', as well as the dialectic pairs of 'reduction versus unification', simplicity versus complexity', and 'local versus global processes'.

These notions are reflections of fundamental biological processes. For example, 'continuity' was already mentioned regarding the relationship between unconscious and conscious processes. Moreover, it was via 'associations' that my unconscious decided of how to greet my wife. The relationship of the other notions with specific biological and neuronal mechanisms will be discussed throughout this volume.

The usefulness of the above concepts will be illustrated by their employment in elucidating aspects of mathematics, physics, biology, neuroscience, medicine, technology, philosophy, and painting. In particular, mathematics and physics provide examples *par excellence* of 'generalization' and 'unification' respectively. Painting provides a plethora of examples of both 'reduction' (as in paintings by Mondrian, Malevich, and Rothko), as well as 'simplicity versus complexity' (as in paintings of Kandinsky, Picasso, and Braque).

The Concrete Outcomes of the New Approach

The integrative approach to knowledge and culture based on this proposed framework has several concrete outcomes, discussed below.

(i) It suggests a way for resolving the paradox between the abundance of information and specialization. In this connection, it is prudent to be aware of the confusion and misinformation generated by the current availability of massive amounts of data. This suggests that 'ignorance' is promoted from two opposite extremes: from too few or from too many data. The relationship between information, knowledge, and wisdom was succinctly expressed by T. S. Elliott: 'Where is the wisdom we have lost in knowledge? Where is the knowledge we have lost in information?'. This volume presents wide and interrelated knowledge. Moreover, it provides *a framework for the interested reader to transform some of the enormous amount of available data and dispersed information into knowledge*. Using the internet and a variety of other sources, material presented in this volume can provide both motivation and guidelines for acquiring additional information regarding a plethora of important areas. Incidentally, I envision a synergistic relationship between this volume and the web: the internet can immediately clarify the meaning of those concepts that I have not defined, erroneously presuming they are known. In addition, the web can be a source of supplementary information for many concrete entities and concepts presented here. For example, one may easily access the history of the paintings analysed in this book.

(ii) It introduces an *all-encompassing approach to sciences and humanities* that will hopefully help restore the arts and letters to their rightful position as the centre of the human existence. The human mental capacity for *computability* gives rise to the enormous computational capabilities of mathematics and the stunning achievements of artificial intelligence (and especially of 'deep learning'). As a result of these achievements, there is nowadays the illusion that algorithms are omnipotent. This illusion has regrettably led to many questioning the importance of arts and letters. Moreover, the arrogant claim that human thought will soon be surpassed by artificial intelligence has led to a distortion of our view of the essence of humanity. However, *mathemat-*

ics reflects only a limited subset of human thought, and artificial algorithms mimic only a small part of the brain processes. Indeed, artificial intelligence not only ignores unconscious processes but also the vital role of the glial cells, the differences in the functions of the two brain's hemispheres, and the crucial fact that the mind is *embodied*. Furthermore, the important *intrinsic limitations* of both mathematics and computing are not taken into serious consideration. In this connection it is noted that, in a similar manner that Ludwig Wittgenstein exposed the limitations of language, the brilliant mathematicians, Kurt Gödel and Alan Turing, rigorously established that *neither mathematics nor computing can reach the truth via a formal, axiomatic process.* In contrast to the current apotheosis of the rational and the algorithmic, the unified, balanced approach advocated in this book suggests that *a basic characteristic of creative ideas and advanced artistic representations is that they are non-verbal and non-algorithmic; they are transcendental. These mental creations are generated via the interaction of unconscious processes and re-representations in the dynamic environment of the embodied brain, which is crucially affected by hormones and other molecules excreted by the body proper.* Hence, I am highly sceptical as to whether such processes can be 'programmed'. Overall, the serious limitations of rationality, together with the pivotal role of the unconscious in arts and letters (that makes these creations even 'less programmable'), imply that in the 21th century, Humanities define what it means to be human *more poignantly than ever before*.

(iii) It demonstrates that *a unified, integrative approach* to knowledge *is* indeed possible. For this purpose, an effort is made to refute the myth that it is supposedly impossible to be both deep and broad. Actually, breadth and depth are not antithetical; they act synergistically. Indeed, *the more areas one is exposed to, the more extensive becomes the web of possible associations among elements and concepts of these diverse areas. Hence, the deeper the insight gained, the higher the appreciation for the value of arts and letters, and the more likely the attainment of difficult goals.* These diverse, positive experiences are accompanied by happiness, or more precisely by *eudemonia;* the state of elation and personal fulfilment achieved via pursuing *knowledge* and *beauty* and *attaining lofty goals*. The word eudemonia is of Greek origin; it derives from the prefix *eu*, meaning good, and *daemon*, meaning spirit. This concept was introduced by the ancient Greeks (in particular, it is discussed in Aristotle's monumental work *Nicomachean Ethics*) and was later elaborated by several scholars. This concept is much broader than *hedonia,* that derives from the Greek word *hēdonē,* meaning pleasure (hedonism, as envisioned by Aristippus, advocates maximizing pleasure). The joy and fulfilment that naturally accompanies the process of approaching the 'essence of things' has been perfectly expressed by Albert Einstein. Recalling the moment when he

conceived the basic idea behind his General Theory of Relativity, Einstein wrote that 'this was the happiest idea of my life'. Perhaps only the unique genius of Einstein, in addition to relating the seemingly far removed notions of gravity and acceleration, was also able to express so eloquently the sequel of any great achievement. He did not characterize his brilliant idea as the most profound or the most original, but simply as the happiest! Many deep thinkers have noted that understanding brings joy. For example, the philosopher Daniel Dennett, states that 'I find comprehension to be one of life's greatest thrills'.[5] The feeling of eudemonia associated with knowledge is consistent with the Aristotelian understanding that 'All men by nature desire knowledge'.[6]

Regarding beauty, it must be emphasized that aesthetic pleasure is not only found in the arts and letters, but also in many other endeavours, including mathematics, science, and technology. I am often mesmerized by music, painting, and poetry; but also by the beauty of several mathematical works. The high aesthetic value of the mathematical equations that express physical laws is perhaps a reflection of the beauty of the corresponding physical reality. Interestingly, there exist common neuronal mechanisms responsible for appreciating different aesthetic forms. For example, a study using functional MRI has shown that, when mathematicians are exposed to musical or visual beauty, there is activation in the same part of their brain as the part activated when they are exposed to a beautiful mathematical equation.[7] The relationship between beauty and mathematics is further discussed in chapter 18.

(iv) The approach introduced in this volume provides the proper framework for an illuminating discussion of several important questions which, in my opinion, should concern every educated individual. They include the following: What is the origin of the distinguishing mental advantages of humans in comparison to our evolutionary predecessors? What is the relationship between innate and acquired knowledge? What does it mean to 'understand' and how is insight achieved? Why is it possible for us to comprehend the universe? What is the effect of the cultural evolution on our brains? What is the neuronal origin of our emotional responses to arts and letters? Could the unbalanced emphasis on science and technology at the expense of arts and humanities 'end up downgrading humans' as Yuval Noah Harari worries in his *Homo Deus*?[8] Can the problem of consciousness be solved?

[5] D.C. Dennett, *From Bacteria to Bach and Back* (Penguin Press, 2017).

[6] '*Πάντες άνθρωποι του ειδέναι ορέγονται φύσει*'.

[7] In this study, there was activation in the medial-orbito-frontal cortex. Among equations considered beautiful were the one expressing Pythagoras' theorem, as well Euler's equation $e^{i\pi}=-1$ [S. Zeki, J.P. Romaya, D.M.T. Benincasa, and M.F. Atiyah 'The experience of mathematical beauty and its neural correlates', in *Front. Hum. Neurosci* 8 (2014)].

[8] Y.N. Harari, *Homo Deus: A Brief History of Tomorrow* (Harvill Secker, 2016) (Original publication: 2015 in Hebrew).

Why is beauty important in those mathematical expressions that capture basic physical phenomena? Can the impact of mathematics in biology be as essential as it has been in physics? The significance of the last question becomes evident by noting the claims of several leading neuroscientists that 'mathematics will be crucial for solving the problem of consciousness'.

(v) Studying this volume will allow the reader to become familiar with many facts that, because of their significance, should be widely known. For example, neuronal mechanisms related to unconscious perception and awareness will be discussed. In addition, it will be shown that the transition from the unconscious to awareness is not only relatively slow but, more importantly, it is accompanied by loss of information. For example, in the so-called 'binocular rivalry', analysed in detail in chapter 14, two *different* images are shown in the left and right visual fields. *Both* images are perceived unconsciously but we become aware of only *one* of them. This clearly shows that *our brains 'know' much more than we do*.

(vi) It is expected that the exposure to different areas of science and humanities will train the brain of young people to *adopt a flexible, multidisciplinary way of thinking*. In my opinion, this provides the best preparation for modern life, where lines between disciplines have become blurred. Most importantly, it will make available to the reader a *new methodology* that can facilitate their analysis of a variety of phenomena. In particular, it will allow them to comprehend the origin of their thoughts, feelings, and actions. This methodology is based on the understanding of the continuity between unconscious and conscious processes, as well as on the judicious employment of the tools mentioned earlier, namely, 'continuity', 'associations', 'interconnectedness', etc. I hope that researchers with various areas of expertise, after reading this volume will be motivated to revisit many disciplines including philosophy, literature, and social sciences, within the proposed methodology. For example, it will be interesting to elucidate how the extensive *network of associations* constructed by the brain motivates the creation of *social networks*. In this connection it is noted that social support, social relations, and friendship, are not only vital components of happiness, but also promote health and affect longevity. People with satisfactory social relationships improve substantially their chance of survival in comparison to those with poor ones.[9]

(vii) The book will expose the reader to elements of biology, neuroscience, medicine, mathematics, and physics in a clear and comprehensible manner. In addition, the reader will be exposed to the stunning recent developments in brain imaging, which allow observation of specific functions of the brain in real time. This achieves a double goal: on the one hand, these elements

[9] J. Holt-Lunstad, T.B. Smith, and J.B. Layton, 'Social relationships and mortality risk: A meta-analytic review', in *PLoS Med*. 7.7 (2010).

provide *illustrations of the novel methodological approach to knowledge advocated in this volume*. At the same time, they offer *a global vista* of these important disciplines and developments. In particular, several chapters present a thorough introduction to neuroscience, from single neurons to various neuronal mechanisms crucial for perception, memory, and learning. In these chapters several important neurological diseases are discussed, and current treatments are noted. In chapter 21 the impact on medicine of mathematics, computer sciences, physics, chemistry, bacteriology, pharmacology, molecular biology, and immunology is discussed. This clearly establishes the interdisciplinary nature of medicine. Moreover, chapter 23 offer the reader extensive and useful information regarding the current very hopeful advances in cancer treatments.

Certainly, the deeper one explores different realms of human endeavours by employing tools elaborated in this volume, the more relationships one discovers. For example, science, with its emphasis on the rational and religion with its reliance on metaphysics, can appear completely antithetical. However, it turns out that, at a deeper level, science and religion, paradoxically, share some common ground. Indeed, taking into consideration the limitations of the rational mentioned earlier, it follows that *the highest aspiration of science coincides with a fundamental goal of religion: it aims to go beyond the rational and to reach the transcendental*. In addition, it is well known that a key element of religion is the belief in a supernatural world. The creation, in the 'soul' of a religious individual, of this invincible, undefined, non-material world, *has its origin in similar neurological mechanisms* that give rise the remote 'echoes' reflecting deep, esoteric, unanswered questions of science, in the brain of a mature scientist. These mechanisms are unknown processes (for the time of writing) that take place in the world of the unconscious. This fundamental characteristic of science was certainly recognized by Einstein, who wrote: 'The most beautiful thing we can experience is the mysterious. It is the source of all true art and science'.

The above remarks suggest that polemical conflicts between science and religion can be avoided. Interestingly, Father and polymath Marin Mersenne (1588–1648), considered 'the cause of science as the cause of God'. Mersenne was a friend of the great scientists Galileo Galilei, Rene Descartes, and Etienne Pascal. He was also in contact with Pierre de Fermat and tried to find a formula that would represent all prime numbers.[10]

It is remarkable that despite the enormous importance of unconscious processes, these processes remain largely undervalued. This is also reflected by the fact that psychiatry, which is the part of medicine mostly concerned with unconscious mech-

[10] Mersenne discovered the 'laws of stretched strings', namely, the formula expressing the frequency of the acoustic waves generated by a stretched string. After his death, his correspondence with Galileo, Fermat, Constantijn Huygens, Evangelista Torricelli, and other scientists, was found in his cell [Tononi 2012].

anisms, is underestimated and underfunded. Psychiatry comes from the Greek words '*psych*ē' and '*iatreia*', which mean 'soul' and 'healing', respectively, and perhaps the best interpretation of the notion of the *soul* is the *unconscious*. Considering the importance and prevalence of psychiatric illnesses, and especially of depression and schizophrenia, this lack of appreciation of the significance of psychiatry is utterly unfortunate. Hopefully, the elucidation of several aspects of the unconscious presented in this volume, will help towards elevating psychiatry to the level it deserves.

According to Aristotle, happiness is 'the end to which our actions are directed'. In my opinion, a crucial component of happiness is *diversity of positive experiences*. Concrete suggestions that may assist young people to achieve such diversity are presented at the end of the book.

In summary, this volume dares to suggest a unifying approach to life that is based on the elucidation of deep neuronal mechanisms, which finally lead to understanding and fulfilment. *Every thought and every activity gives rise to a multitude of associations and hence (if appropriately manipulated) to the potential for generating the feeling of eudemonia.*